

HOT NONPERTURBATIVE QCD*

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Abstract

Starting from the new background field formalism for $T > 0$, with nonperturbative (NP) background given as gauge invariant field correlators, new perturbation theory and diagram technic is introduced.

Confined and deconfined phases are explicitly described and critical temperature T_c is expressed in terms of the scale anomaly term. Resulting numerical estimates of T_c agree well with lattice data. Spatial area law is shown to follow naturally for $T > T_c$, and a set of Hamiltonians is obtained for screening masses of mesons and glueballs depending on the Matsubara frequencies.

The lowest screening masses and wave functions of mesons obtained agree with lattice data and earlier calculations. Screening glueball masses and wave functions are also computed from the Hamiltonian and two different regimes are observed in high and low T regions.

The infrared catastrophe of hot QCD is shown to be cured by the NP background.

1 Introduction

The finite temperature QCD provides a unique insight into the structure of the QCD vacuum, in particular it gives an important information about the mechanism of confinement. It is also of more practical interest, since the

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nature of the deconfinement transition has its bearing on the cosmology and the hot QCD plasma can possibly be tested in heavy ion collisions.

From theoretical point of view the hot QCD is a unique theoretical laboratory where both perturbative (P) and nonperturbative (NP) methods can be applied in different temperature regimes.

It is believed that the perturbative QCD is applicable in the deconfined phase at large enough temperatures T , where the effective coupling constant $g(T)$ is small [1], while at small T (in the confined phase) the NP effects instead are most important. However even at large T the physics is not that simple: some effects, like screening (electric gluon mass), need a resummation of the perturbative series [2], while the effects connected with the magnetic gluon mass demonstrate the infrared divergence of the series [3].

During last years there appeared a lot of lattice data which point at the NP character of dynamics above T_c . Here belong i) area law of spacial Wilson loops [4] ii) screening masses of mesons and baryons [5] and glueballs [6] iii) temperature dependence of Polyakov–line correlators [7].

In addition, behaviour of $\varepsilon - 3p$ above T_c has a bump incompatible with the simple quark–gluon gas picture [8].

Thus the inclusion of NP configurations into QCD at $T > 0$ and also at $T > T_c$ is necessary.

Recently a systematic method for QCD was developed, treating NP fields as a background and doing perturbative expansion around that both for $T = 0$ [9] and $T > 0$ [10].

To describe the phase transition a simple choice of deconfined phase was suggested where all NP color magnetic configurations are kept intact as in the confined phase, whereas colorelectric correlators responsible for confinement, vanish.

This picture together with the background perturbation theory forms a basis of quantitative calculations, where field correlators (condensates) are used as the NP input.

The plan of the lecture is as follows. In the second chapter the new background field formalism is presented, based on the familiar background field method [11] modified for $T > 0$ and incorporating the 'tHooft's identity for integration over quantal and background fields.

The temperature phase transition is discussed in chapter 3 and resulting predictions for T_c are compared with lattice data.

In chapter 4 the spacial Wilson loops are computed both below and above

T_c ; it is shown how at large T a new regime – that of dimensional reduction – appears and the spacial string tension is discussed.

Chapter 5 is devoted to screening masses of mesons, and glueballs, which were considered recently in [5,6].

In chapter 6 the analysis is given of the infrared catastrophe of the hot QCD; it is shown, that it is cured naturally in the present formalism by nonperturbative contributions.

Some prospectives of the method and discussion of results are given in Conclusions.

2 New background field formalism

We derive here basic formulas for the partition function, free energy and Green's function in the NP background formalism at $T > 0$ [10]. The total gluonic field A_μ is split into a perturbative part a_μ and NP background B_μ

$$A_\mu = B_\mu + a_\mu \quad (1)$$

where both B_μ and a_μ are subject to periodic boundary conditions. The principle of this separation is immaterial for our purposes here, and one can average over fields B_μ and a_μ independently using the 'tHooft's identity¹

$$Z = \int DA_\mu \exp(-S(A)) = \frac{\int DB_\mu \eta(B) \int Da_\mu \exp(-S(B+a))}{\int DB_\mu \eta(B)} \quad (2)$$

$$\equiv \langle \langle \exp(-S(B+a)) \rangle_a \rangle_B$$

with arbitrary weight $\eta(B)$. In our case we choose $\eta(B)$ to fix field correlators and string tension at their observed values.

The partition function can be written as

$$Z(V, T, \mu = 0) = \langle Z(B) \rangle_B ,$$

$$Z(B) = N \int D\phi \exp(-\int_0^\beta d\tau \int d^3x L(x, \tau)) \quad (3)$$

¹private communication to one of the authors (Yu.S.), December 1993.

where ϕ denotes all set of fields a_μ, Ψ, Ψ^+, N is a normalization constant, and the sign $\langle \rangle_B$ means some averaging over (nonperturbative) background fields B_μ , as in (2). Furthermore, we have

$$L(x, \tau) = \sum_{i=1}^8 L_i,$$

where

$$\begin{aligned} L_1 &= \frac{1}{4}(F_{\mu\nu}^a(B))^2; L_2 = \frac{1}{2}a_\mu^a W_{\mu\nu}^{ab} a_\nu^b, \\ L_3 &= \bar{\Theta}^a (D^2(B))_{ab} \Theta^b; L_4 = -ig \bar{\Theta}^a (D_\mu, a_\mu)_{ab} \Theta^b \\ L_5 &= \frac{1}{2} \alpha (D_\mu(B) a_\mu)^2; L_6 = L_{int}(a^3, a^4) \\ L_7 &= -a_\nu D_\mu(B) F_{\mu\nu}(B); L_8 = \Psi^+ (m + \hat{D}(B + a)) \Psi \end{aligned} \quad (4)$$

Here $\bar{\Theta}, \Theta$ are ghost fields, α - gauge-fixing constant, L_6 contains 3-gluon- and 4-gluon vertices, and we keep the most general background field B_μ , not satisfying classical equations, hence the appearance of L_7 .

The inverse gluon propagator in the background gauge is

$$W_{\mu\nu}^{ab} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2g F_{\mu\nu}^c(B) f^{acb} \quad (5)$$

where

$$(D_\lambda)_{ca} = \partial_\lambda \delta_{ca} - ig T_{ca}^b B_\lambda^b \equiv \partial_\lambda \delta_{ca} - g f_{bca} B_\lambda^b \quad (6)$$

We consider first the case of pure gluodynamics, $L_8 \equiv 0$.

Integration over ghost and gluon degrees of freedom in (3) yields

$$\begin{aligned} Z(B) &= N' (det W(B))_{reg}^{-1/2} [det(-D_\mu(B) D_\mu(B + a))]_{a=\frac{\delta}{\delta J}} \times \\ &\times \{1 + \sum_{l=1}^{\infty} \frac{S_{int}^l}{l!} (a = \frac{\delta}{\delta J})\} exp(-\frac{1}{2} J W^{-1} J)_{J_\mu = D_\mu(B) F_{\mu\nu}(B)} \end{aligned} \quad (7)$$

One can consider strong background fields, so that gB_μ is large (as compared to Λ_{QCD}^2), while $\alpha_s = \frac{g^2}{4\pi}$ in that strong background is small at all distances [9].

In this case Eq. (7) is a perturbative sum in powers of g^n , arising from expansion in $(ga_\mu)^n$.

In what follows we shall discuss the Feynman graphs for the free energy $F(T)$, connected to $Z(B)$ via

$$F(T) = -T \ln \langle Z(B) \rangle_B \quad (8)$$

As will be seen, the lowest order graphs already contain a nontrivial dynamical mechanism for the deconfinement transition, and those will be considered in the next section.

To the lowest order in ga_μ the partition function and free energy are

$$\begin{aligned} Z_0 &= \langle \exp(-F_0(B)/T) \rangle_B, \\ F_0(B)/T &= \frac{1}{2} \ln \det W - \ln \det(-D^2(B)) = \\ &= Sp \int_0^\infty \zeta(t) \frac{dt}{t} \left(-\frac{1}{2} e^{-tW} + e^{tD^2(B)} \right) \end{aligned} \quad (9)$$

where $\hat{W} = -D^2(B) - 2g\hat{F}$ and $D^2(B)$ is the inverse gluon and ghost propagator respectively, $\zeta(t)$ is a regularizing factor [10].

The ghost propagator can be written as [10],[12]

$$(-D^2)_{xy}^{-1} = \langle x | \int_0^\infty dt e^{tD^2(B)} | y \rangle = \int_0^\infty dt (Dz)_{xy}^w e^{-K} \Phi(x, y) \quad (10)$$

where standard notations [12] are used

$$K = \frac{1}{4} \int_0^s d\lambda \dot{z}_\mu^2, \quad \Phi(x, y) = P \exp i g \int_y^x B_\mu dz_\mu$$

and a winding path integral is introduced [10]

$$(Dz)_{xy}^w = \lim_{N \rightarrow \infty} \prod_{m=1}^N \frac{d^4 \zeta(m)}{(4\pi\epsilon)^2} \sum_{n=0, \pm 1, \dots}^\infty \int \frac{d^4 p}{(2\pi)^4} e^{ip(\sum \zeta(m) - (x-y) - n\beta\delta_{\mu 4})} \quad (11)$$

with $\beta = 1/T$. For the gluon propagator an analogous expression holds true, expect that in (4) one should insert gluon spin factor $P_F \exp 2g\hat{F}$ inside $\Phi(x, y)$. For a quark propagator the sum over windings in (5) acquires the factor $(-1)^n$ and quark spin factor is $\exp g \sigma_{\mu\nu} F_{\mu\nu}$ [10].

3 The temperature phase transition in QCD

We are now in position to make expansion of Z and F in powers of ga_μ (i.e. perturbative expansion in α_s), and the leading–nonperturbative term Z_0, F_0 – can be represented as a sum of contributions with different N_c behaviour of which we systematically will keep the leading terms $0(N_c^2), 0(N_c)$ and $0(N_c^0)$.

To describe the temperature phase transition one should specify phases and compute free energy. For the confining phase to lowest order in α_s free energy is given by Eq.(3) plus contribution of energy density ε at zero temperature

$$F(1) = \varepsilon V_3 - \frac{\pi^2}{30} V_3 T^4 - T \sum_s \frac{V_3 (2m_s T)^{3/2}}{8\pi^{3/2}} e^{-m_s/T} + 0(1/N_c) \quad (12)$$

where ε is defined by scale anomaly [13]

$$\varepsilon \simeq -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} < (F_{\mu\nu}^a(B))^2 > \quad (13)$$

and the next terms in (12) correspond to the contribution of mesons (we keep only pion gas) and glueballs. Note that $\varepsilon = 0(N_c^2)$ while two other terms in (12) are $0(N_c^0)$.

For the second phase (to be the high temperature phase) we make an assumption that there all color magnetic field correlators are the same as in the first phase, while all color electric fields vanish. Since at $T = 0$ color–magnetic correlators (CMC) and color–electric correlators (CEC) are equal due to the Euclidean $0(4)$ invariance, one has

$$< (F_{\mu\nu}^a(B))^2 > = < (F_{\mu\nu}^a)^2 >_{el} + < (F_{\mu\nu}^a)^2 >_{magn}; < F^2 >_{magn} = < F^2 >_{el} \quad (14)$$

The string tension σ which characterizes confinement is due to the electric fields [14], e.g. in the plane (i4)

$$\sigma = \sigma_E = \frac{g^2}{2} \int \int d^2x < tr E_i(x) \Phi(x, 0) E_i(0) \phi(0, x) > + \dots \quad (15)$$

where dots imply higher order terms in E_i .

Vanishing of σ_E liberates gluons and quarks, which will contribute to the free energy in the deconfined phase their closed loop terms (10) with all

possible windings. The CMC enter via perimeter contribution $\langle \Phi(x, x) \rangle \equiv \Omega$ (see (9,10)). As a result one has for the high-temperature phase (phase 2) (cf.[10]).

$$F(2) = \frac{1}{2}\varepsilon V_3 - (N_c^2 - 1)V_3 \frac{T^4 \pi^2}{45} \Omega_g - \frac{7\pi^2}{180} N_c V_3 T^4 n_f \Omega_q + 0(N_c^0) \quad (16)$$

where Ω_q and Ω_g are perimeter terms for quarks and gluons respectively, the latter was estimated in [15] from the adjoint Polyakov line; in what follows we replace Ω by one for simplicity.

Comparing (12) and (16), $F(1) = F(2)$ at $T = T_c$, one finds in the order $0(N_c)$, disregarding all meson and glueball contributions

$$T_c = \left(\frac{\frac{11}{3} N_c \frac{\alpha_s \langle F^2 \rangle}{32\pi}}{\frac{2\pi^2}{45} (N_c^2 - 1) + \frac{7\pi^2}{90} N_c n_f} \right)^{1/4} \quad (17)$$

For standard value of $G_2 \equiv \frac{\alpha_s}{\pi} \langle F^2 \rangle = 0.012 \text{GeV}^4$ [13] (note that for $n_f = 0$ one should use approximately 3 times larger value of G_2 [13]) one has for $SU(3)$ and different values of $n_f = 0, 2, 4$ respectively $T_c = 240, 150, 134$ MeV. This should be compared with lattice data [8] $T_c(\text{lattice}) = 240, 146, 131$ MeV. Agreement is quite good. Note that at large N_c one has $T_c = 0(N_c^0)$ i.e. the resulting value of T_c doesn't depend on N_c in this limit. Hadron contributions to T_c are $0(N_c^{-2})$ and therefore suppressed if T_c is below the Hagedorn temperature as it typically happens in string theory estimates [16].

Till now we disregarded all perturbative and nonperturbative corrections to $F(2)$ except for magnetic condensate, the term $\frac{1}{2}\varepsilon V_3$. If we disregard also this term, considering in this way only free gas of gluons and quarks for the phase 2, we come to the model, considered in [17]. The values of T_c obtained in this way differ from ours not much – they are factor of $2^{1/4} = 1.19$ larger, but one immediately encounters problems with explanation of spacial string tension, screening masses etc., which are naturally accounted for by the notion of magnetic confinement – nonzero values of magnetic correlators in the phase 2, including magnetic condensate term, $\frac{1}{2}\varepsilon V_3$.

Our approximation (10) corresponding to lowest order in N_C and g was too simplified when we have put $\Omega_g = \Omega_q = 1$.

Indeed NP corrections may contribute to Ω_g, Ω_q . Their phenomenological necessity can be seen in the measured values of $\varepsilon - 3p$, [7,8], which are seen

in Fig.1 In case of $\Omega_g = \Omega_q = 1$ the difference $\varepsilon - 3p$ should be zero, and of course higher orders in N_C^{-1} (NP effects) and higher orders in g (Perturbative effects) contribute to it. In [15] we have tried to estimate effect of nonzero $(\Omega_g - 1)$, which is $O(N_C^2)$, on the energy density and preassure. To this end we exploit the adjoint Polyakov line and separate from it the NP perimeter contribution. This can be done if one subtracts properly the linear divergent perturbative contribution, specific for Wilson and Polyakov contours. Thus one can write [15]

$$\Omega_g(NP) = \frac{\Omega_g(lattice)}{\Omega_g(pert)}, \quad \Omega_g(pert) = \exp\left[-\frac{N_c^2}{\beta_L} G(0) N_c\right] \quad (18)$$

Substituting this value of $\Omega_g(NP)$ into $F(2)$, eq.(16), one obtains ε and p , shown in Fig.2 by the line. One can see a reasonable agreement between this estimate and lattice data, which could signify the importance of gluon perimeter contribution. That was done for the $SU(2)$ group, since only there exist detailed lattice data for the adjoint Polyakov line.

One can now estimate the influence of nonzero $\Omega_g - 1$ on T_C , again for the $SU(2)$ case.

Taking $\Omega_g(NP)$ from (18) and substituting it into (17) one obtains a shift of T_C by a factor of 1.07 for $n_f = 2$ [18]. Similar calculations for $SU(3)$ are now in progress [18].

4 Spacial Wilson loops

In this section we derive area law for spacial Wilson loops, expressing spacial string tension in terms of CMC.

To this end we write $\langle W(C) \rangle$ for any loop as [14]

$$\begin{aligned} \langle W(C) \rangle = \exp\left[-\frac{g^2}{2} \int d\sigma_{\mu\nu}(u) d\sigma_{\rho\lambda}(u') \ll F_{\mu\nu}(u) \Phi(u, u') F_{\rho\lambda}(u') \Phi(u', u) \gg \right. \\ \left. + \text{higher order cumulants} \right] \end{aligned} \quad (19)$$

For temporal Wilson loops, in the plane $i4, i = 1, 2, 3$, only color electric fields $E_i = F_{i4}$ enter in (19), while for spacial ones in the plane $i, k; i, k = 1, 2, 3$ there appear color magnetic field $B_i = \frac{1}{2} \epsilon_{ikl} F_{kl}$; in standard way [14]

one obtains the area law for large Wilson loops of size L , $L \gg T_g^{(m)}$ ($T_g^{(m)}$ is the magnetic correlation length)

$$\langle W(C) \rangle_{spacial} \approx \exp(-\sigma_s S_{\min}) \quad (20)$$

where the spacial string tension is [10,14]

$$\sigma_s = \frac{g^2}{2} \int d^2x \ll B_n(x) \Phi(x, 0) B_n(0) \Phi(0, x) \gg + 0(\langle B^4 \rangle) \quad (21)$$

and n is the component normal to the plane of the contour, while the last term in (21) denotes contribution of the fourth and higher order cumulants. On general grounds one can write for the integrand in (21)

$$\ll B_i(x) \Phi(x, 0) B_j(0) \Phi(0, x) \gg = \delta_{ij} (D^B(x) + D_1^B(x) + \vec{x}^2 \frac{\partial D_1^B}{\partial x^2}) - x_i x_j \frac{\partial D_1^B}{\partial x^2}, \quad (22)$$

and only the term $D^B(x)$ enters in (21)

$$\sigma_s = \frac{g^2}{2} \int d^2x D^B(x) + 0(\langle B^4 \rangle) \quad (23)$$

similarly for the temporal Wilson loop in the plane $i4$ one has the area law for $T < T_c$ with temporal string tension

$$\sigma_E = \frac{g^2}{2} \int d^2x D^E(x) + 0(\langle E^4 \rangle) \quad (24)$$

For $T = 0$ due to the $0(4)$ invariance CEC and CMC coincide and $\sigma_E = \sigma_s$. For $T > T_c$ in the phase (2) CEC vanish, while CMC change on the scale of the dilaton mass $\sim 1\text{GeV}$, therefore one expects that σ_s stays intact till the onset of the dimensional reduction mechanism. This expectation is confirmed by the lattice simulation – σ_s stays constant up to $T \approx 1.4T_c$ [19]. Recent lattice data [19] show an increase of σ_s at $T \approx 2T_c$, for $SU(2)$ which could imply the early onset of dimensional reduction.

Indeed the numerical analysis [7] shows that at $T \geq 2T_c$ the spacial string tension σ_s can be well reproduced as

$$\sqrt{\sigma_s(T)} = cg^2(T)T \quad (SU_3) \quad (25)$$

which should be compared with the 3d QCD value [20]

$$\sqrt{\sigma_s} = 0.554g_3^2 \quad (SU_3) \quad (26)$$

The constant c in (25) is actually expandable as a series of g^n , but in the range $2 \leq T/T_c \leq 4$ it is approximately constant, $c \simeq 0.63$. Comparison of (25) and (26) with $g_3^2 = g^2(T)T$ indeed supports an early dimensional reduction with the coupling constant in the 2-loop approximation

$$\frac{1}{g^2(T)} = b_0 \ln\left(\frac{T}{\Lambda_T}\right)^2 + \frac{b_1}{b_0} \ln \ln\left(\frac{T}{\Lambda_T}\right)^2 \quad (27)$$

where $b_0 = \frac{11}{16\pi^2}$, $b_1 = \frac{102}{(16\pi^2)^2}$ and the value Λ_T fitted to $\sigma_s(T)$ is

$$\Lambda_T^\sigma = (0.076 \pm 0.013)T_c \quad (28)$$

Due to the small value of Λ_T , $g(T)$ is also small, $g(2T_c) \approx \sqrt{2}$ as compared to the $4d, T = 0$ value from heavy quarkonia, $\Lambda \approx 200 MeV$.

Question: Why dimensional reduction sets in as early as $T = 2T_c$?

5 Screening masses of mesons and glueballs

This section is based on results of ref. [21]. In this section we consider the $q\bar{q}$ and gg Green's functions $G(x, y)$ at $T > T_c$ and derive corresponding Hamiltonians for evolution in the spacial direction. We start with the Feynman-Schwinger representation [22] for $G(x, y)$, where we neglect for simplicity spin interaction terms

$$G(x, y) = \int_0^\infty ds \int_0^\infty d\bar{s} e^{-K-\bar{K}} (Dz)_{xy}^w (D\bar{z})_{xy}^w < W(C) > \quad (29)$$

Here K and $(Dz)_{xy}^w$ are defined in (10) and $< W(C) >$ in (20), where the contour C is formed by paths $z(\tau)$, $\bar{z}(\tau)$ and $t \equiv x - y$ is for definiteness along the axis 3. Since by definition at $T > T_c$ electric correlators are zero, only elements $d\sigma_{\mu\nu}$ in (19) in planes 12, 13 and 23 contribute. As a result one obtains for $< W(C) >$ the form (20) with

$$S_{min} = \int_0^t d\tau \int_0^1 d\gamma \sqrt{\dot{w}_i^2 w_k'^2 - (\dot{w}_i w_i')^2} \quad (30)$$

where only spacial components $w_i, i = 1, 2, 3$ enter

$$w_i(\tau, \gamma) = z_i(\tau)\gamma + \bar{z}_i(\tau)(1 - \gamma), \dot{w}_i = \frac{\partial w_i}{\partial \tau}, \quad w'_i = \frac{\partial w_i}{\partial \gamma} \quad (31)$$

The form (30) is equivalent to that used before in [12] but with $w_4 \equiv 0$.

As a next step one can introduce "dynamical mass" $\mu, \bar{\mu}$ similarly to [12]. We are looking for the "c.m" Hamiltonian which corresponds to the hyperplane where $z_3 = \bar{z}_3$. Now the role of evolution parameter (time) is played by $z_3 = \bar{z}_3 = \tau$ with $0 \leq \tau \leq t$, and we define transverse vectors $z_\perp(z_1, z_2), \bar{z}_\perp(\bar{z}_1, \bar{z}_2)$ and $z_4(\tau), \bar{z}_4(\tau)$.

$$\frac{dz_3}{d\lambda} = \frac{d\tau}{d\lambda} = 2\mu, \quad \frac{d\bar{z}_3}{d\bar{\lambda}} = \frac{d\tau}{d\bar{\lambda}} = 2\bar{\mu}, \quad (32)$$

then K, \bar{K} in (29) assume the form

$$K = \frac{1}{2} \int_0^t d\tau \left[\frac{m_1^2}{\mu(\tau)} + \mu(\tau)(1 + \dot{z}_\perp^2 + \dot{z}_4^2) \right] \quad (33)$$

and the same for \bar{K} with additional bars over $\mu, \dot{z}_\perp, \dot{z}_4$.

Performing the transformation in the functional integral (29) $dsDz_3(\tau) \rightarrow D\mu, \quad d\bar{s}D\bar{z}_3(\tau) \rightarrow D\bar{\mu}$ one has

$$G(x, y) = \int D\mu D\bar{\mu} Dz_\perp D\bar{z}_\perp (Dz_4)_{xy}^w (D\bar{z}_4)_{xy}^w \exp(-A) \quad (34)$$

with the action

$$A = K + \bar{K} + \sigma S_{min} \quad (35)$$

Note that z_4, \bar{z}_4 are not governed by NP dynamics and enter A only kinematically (through K, \bar{K}), and hence can be easily integrated out in (34) using Eq.(11) for the 4-th components – with $(x - y)_4 = 0$. One can now proceed as it was done in [12], i.e. one introduces auxiliary functions $\nu(\tau, \gamma), \eta(\tau, \gamma)$; defines center-of-mass and relative coordinates $\vec{R}_\perp, \vec{r}_\perp \equiv \vec{r}$, and finally integrates out \vec{R}_\perp and $\eta(\tau, \gamma)$. The only difference from [12] is that now z_4, \bar{z}_4 do not participate in all those transformations. As a result one obtains

$$G(x, y) \sim \int D\nu D\mu D\bar{\mu} D\tau e^{-A^{(1)}} \sum_{n, n_2} e^{-A_{n_1 n_2}} \quad (36)$$

here

$$A^{(1)}(\mu, \bar{\mu}, \nu) = \frac{1}{2} \int_0^t d\tau \left[\frac{\vec{p}^2 + m_1^2}{\mu} + \frac{\vec{p}^2 + m_2^2}{\bar{\mu}} + \mu + \bar{\mu} + \sigma^2 r^2 \int_0^1 \frac{d\gamma}{\nu} + \int_0^1 \nu d\gamma + \right. \quad (37)$$

$$\left. + \frac{\vec{L}^2/r^2}{\mu(1-\zeta)^2 + \bar{\mu}\zeta^2 + \int_0^1 d\gamma(\gamma-\zeta)^2\nu} \right] \\ A_{n_1 n_2} = \frac{1}{2}(\pi T)^2 \int_0^t d\tau \left(\frac{b^2(n_1)}{\mu(\tau)} + \frac{b^2(n_2)}{\bar{\mu}(\tau)} \right), \quad (38)$$

$b(n) = 2n$ for bosons and $2n + 1$ for quarks. We also have introduced radial momentum \vec{p}_r , angular momentum \vec{L}

$$\vec{p}_r^2 \equiv \frac{(\vec{p}\vec{r})^2}{r^2} = \left(\frac{\mu\bar{\mu}}{\mu + \bar{\mu}} \right)^2 \frac{(\vec{r}\vec{r})^2}{r^2}, \quad \vec{L} = \vec{r} \times \vec{p} \quad (39)$$

and

$$\zeta(\tau) = \frac{\mu(\tau) + \langle \gamma \rangle \int \nu d\gamma}{\mu + \bar{\mu} + \int \nu d\gamma}, \quad \langle \gamma \rangle \equiv \frac{\int \gamma \nu d\gamma}{\int \nu d\gamma} \quad (40)$$

Let us define the Hamiltonian H for the given action $A = A^{(1)} + A_{n_1 n_2}$ in (36), integrating over $D\nu, D\mu, D\bar{\mu}$ around the extremum of A (this is an exact procedure in the limit $t \rightarrow \infty$). For the extremal values of auxiliary fields one has

$$\frac{\vec{p}^2 + m_1^2 + (b(n_1)\pi T)^2}{\mu^2(\tau)} = 1 - \frac{l(l+1)}{\vec{r}^2} \left(\frac{(1-\zeta)^2}{a_3^2} - \frac{1}{\mu^2} \right) \\ \frac{\vec{p}^2 + m_2^2 + (b(n_2)\pi T)^2}{\bar{\mu}^2(\tau)} = 1 - \frac{l(l+1)}{\vec{r}^2} \left(\frac{\zeta^2}{a_3^2} - \frac{1}{\bar{\mu}^2} \right) \quad (41) \\ \frac{\sigma^2}{\nu^2(\tau, \gamma)} \vec{r}^2 = 1 - \frac{l(l+1)}{\vec{r}^2} \frac{(\gamma - \zeta)^2}{a_3^2}$$

where $a_3 = \mu(1-\zeta)^2 + \bar{\mu}\zeta^2 + \int d\gamma(\gamma-\zeta)^2\nu$ and ζ is defined by eq.(40).

After the substitution of these extremal values into the path integral Hamiltonian

$$G(x, y) = \langle x | \sum_{n_1 n_2} e^{-H_{n_1 n_2} t} | y \rangle \quad (42)$$

one has to construct (performing proper Weil ordering) the operator Hamiltonian acting on the wave functions.

Consider for simplicity the case $\vec{L} = 0$, then from (37-38) one obtains

$$H_{n_1 n_2} = \sqrt{\vec{p}^2 + m_1^2 + (b(n_1)\pi T)^2} + \sqrt{\vec{p}^2 + m_2^2 + (b(n_2)\pi T)^2} + \sigma r \quad (43)$$

Here $\vec{p} = \frac{1}{i} \frac{\partial}{\partial \vec{r}}$ and $\vec{r} = \vec{r}_\perp$ is a $2d$ vector, $\vec{r} = (r_1, r_2)$; m_1, m_2 – current masses of quark and antiquark, for gg system $m_1 = m_2 = 0$ and $\sigma = \sigma_{adj} = \frac{9}{4}\sigma$.

Eigenvalues and eigenfunctions of $H_{n_1 n_2}$

$$H_{n_1 n_2} \psi(r) = M(n_1, n_2) \psi(r) \quad (44)$$

define the so-called screening masses and corresponding wave-functions, which have been measured in lattice calculations [5,6,23].

The lowest mass sector for mesons is given by H_{00} , where for $n_1 = n_2 = 0$ one has $b(n_1) = b(n_2) = 1$ in (43). For light quarks one can put $m_1 = m_2 = 0$ and expand at large T square roots in (43) to obtain

$$H_{00} \approx 2\pi T + \frac{\vec{p}^2}{\pi T} + \sigma r, \quad M_{00} \approx 2\pi T + \varepsilon(T) \quad (45)$$

where $\varepsilon(T) = \frac{(\sigma_s(T))^{2/3}}{(\pi T)^{1/3}} a$ and $a \simeq 1.74$ is the eigenvalue of the 2-D dimensionless Schrödinger equation.

Assuming the parametrization $\sqrt{\sigma_s} = c g^2(T) T$ with $c = 0.369$ and scaling behaviour of $g^2(T)$ [19] one has $M_{00} \approx 2\pi T + 0((\ln(T/\Lambda_T))^{-4/3} T)$ tending to twice the lowest Matsubara frequency. (This limit corresponds to the free quarks, propagating perturbatively in the space-time with the imposed antiperiodic boundary conditions along the 4th axis).

Eq. (45) coincides with that proposed in [24], where also numerical study was done of M_{00} and $\psi_{00}(r)$. Our calculations of Eq.(44-45) agree with [24] and are presented in Fig.1 together with lattice calculations of $\psi_{00}(r)$ for ρ – meson [23]. The values of $M_{00}(T)$ found on the lattice [3,4] are compared with our results in Fig.4. Note, that our M_{00} (45) does not contain perimeter corrections which are significant. Therefore one has to add the meson constant to M_{00} to compare with lattice data. We disregard spin-dependent and one-gluon-exchange (OGE) interaction here for lack of space. It is known [24] however, that OGE is not much important at around $T \approx 2T_C$.

We note, that the lowest meson screening mass appear also in Yukawa type exchanges between quark lines and can be compared with the corresponding Yukawa potentials.

For the gg system the lowest mass sector is given by $b(n=0)=0$, and one has from (43)

$$H_{00}(gg) = 2|\vec{p}| + \sigma_{adj}r \quad (46)$$

Note that T does not enter the kinetic terms of (46).

To calculate with (46) one can use the approximation in (37) of τ -independent μ [12], which leads to the operator ($\mu = \bar{\mu}$)

$$h(\mu) = \mu + \frac{\vec{p}^2}{\mu} + \sigma_{adj}r \quad (47)$$

The eigenvalue $E(\mu)$ of $h(\mu)$ should be minimized with respect to μ and the result $E(\mu_0)$ is known to yield eigenvalue of (46) within few percent accuracy [25]. The values $E(\mu_0) \approx M_{gg}$ thus found are presented in Fig.4. The corresponding wave functions $\psi_{00}(r)$ are given in Fig.3. These data can be compared with the glueball screening masses, found on the lattice in [6].

Another point of comparison is Polyakov line correlator

$$P(R) = \langle \Omega(R)\Omega(0) \rangle - \langle \Omega(R) \rangle \langle \Omega(0) \rangle = \exp(-V(R)/T) - 1 \quad (48)$$

with

$$V(R) = \frac{\exp(-\mu R)}{R^\alpha}, \quad \Omega(R) = \frac{1}{N_C} \text{tr} P \exp i g \int_0^\beta A_4 dz_4$$

It is easy to understand that $\mu = M_{gg}$ and one can compare our results for glueball screening mass (GSM) $M_{gg}(T)$ with the corresponding lattice data for μ in Fig.4.

Taking again into account an unknown constant perimeter correction to our values of M_{gg} , one can see a reasonable qualitative behaviour.

In addition to the purely nonperturbative source of GSM described above there is a competing mechanism – the perturbative formation of the electric Debye screening mass $m_{el}(T) = gT(\frac{N_c}{3} + \frac{n_f}{6})$ for each gluon with $g(T)$ given by the temperature dependence of $\sqrt{\sigma_s(T)} = cg^2(T)T$ [7,19]. Therefore for large T , where m_{el} is essential, one should rather use instead of (46) another Hamiltonian, which is obtained from (43) replacing $m_1 = m_2 = m_{el}(T)$

$$\tilde{H}_{00}(gg) = 2\sqrt{\vec{p}^2 + m_{el}^2(T)} + \sigma_{adj}(T)r \quad (49)$$

It turns out that up to the temperatures $T \leq 2T_c$ one can consider the effect of $m_{el}(T)$ as a small correction to the eigenvalue of (46) E

$$M_{gg} = E + \delta E \approx 4\left(\frac{a}{3}\right)^{3/4} \cdot \frac{3}{2}cg^2(T)T + \frac{T}{\frac{3}{2}c\left(\frac{a}{3}\right)^{3/4}}, \quad (50)$$

and that gives us the right to treat GSM in the temperature region concerned nonperturbatively.

We note that the transition between these two regimes (with the dominance of nonperturbative and then perturbative dynamics) is smooth, with GSM tending to twice the Debye mass at large T .

6 Infrared catastrophe in Hot QCD

The infrared (IR) catastrophe in QCD was indentified within the hot perturbative series in [3]. For our purposes we shall explain it in the language of Feynman diagrams in the configuration space. Consider the Feynman diagram for the thermodynamic potential in the order g^n , shown in Fig.5. It can be written as

$$J^{(n)} = \prod_{i=1}^n d^4x^{(i)} \Gamma^{(i)} \prod_{i,j} G(x^{(i)}, x^{(j)}) \quad (51)$$

where G_{ij} is the x -space gluon Green's function connecting vertices i and j , and $\Gamma^{(i)}$ is the three-gluon vertex containing one derivative, $\partial/\partial x^{(i)}$. We suppress all color and Lorentz indices for the sake of brevity and will only count the powers of $x^{(k)}$ to judge the IR convergence of $J^{(n)}$.

At $T > 0$ using Matsubara formalism one has to replace

$$d^4x \rightarrow T^{-1}d^3x$$

$$G(x, y) \rightarrow (\partial^2)_{xy}^{-1} = \sum_{k=0, \pm 1, \dots} \int \frac{T d^3p}{(2\pi)^3} \frac{e^{-i\vec{p}(\vec{x}-\vec{y}) - i2\pi kT(x_4-y_4)}}{(\vec{p}^2 + (2\pi kT)^2)} \quad (52)$$

For large T the only mode propagating over distances larger than $1/T$, as follows from (52) is the mode with $k = 0$. Exactly this mode causes IR catastrophe; we have for it

$$G(x, y) \sim \frac{T}{4\pi|\vec{x}-\vec{y}|}, \quad |\vec{x}-\vec{y}| \gg 1/T \quad (53)$$

Now for the diagram of the type of Fig.(5) one has for a given number of vertices n total number of propagators $\frac{3n}{2}$, so that one can write (remembering that one vertex yields overall volume)

$$J^{(n)} \sim g^n V_3 T^{-n} \frac{(d^3 x)^{n-1} \cdot T^{\frac{3n}{2}}}{x^{\frac{5n}{2}}} \sim \quad (54)$$

$$\sim g^n V_3 T^{\frac{n}{2}} \int (x^{\frac{n}{2}-3})$$

where the the symbol $\int(x^k)$ denotes the overall power of $x^{(i)}$ in the integrand including those of $d^3 x^{(i)}$. One can immediately see that for $n \geq 6$ the diagram $J^{(n)}$ diverges at large x .

We shall now give simple arguments which show that accounting for the nonperturbative background, the perturbation series for F behaves well and IR catastrophe disappears.

To this end we consider the perturbation theory given by Eqs. (1-11).

The diagram of Fig.(5) goes over into the equivalent diagram of the order g^n , where vertices and propagators are now taken in the NP background field.

$$G(x, y) \rightarrow \begin{cases} (-D^2)_{xy}^{-1}, & \text{Eq.(10) for the ghost} \\ (W)_{xy}^{-1} = (-D^2 - 2gF)_{xy}^{-1}, & \text{for the gluon} \end{cases} \quad (55)$$

$$\Gamma^{(i)} \sim \partial/\partial x_\mu^{(i)} \rightarrow \frac{\partial}{\partial x_\mu^{(i)}} - igB_\mu(x^{(i)})$$

The expression for the diagram, $\tilde{J}^{(n)}$ looks the same as (53) with eq.(10), for propagators, but now two major changes appear:

i) All phase factors $\phi(x, y)$ of gluon (ghost) Green's function and vertexes Γ assemble into gauge-invariant combinations of Wilson loops and covariant derivatives, so that the diagram is gauge-invariant.

ii) Averaging over background fields as in (2) brings about Wilson-loop averages, which are subject to area law for large loops (as can be seen from cluster expansion [14] or lattice calculations [19]).

Therefore it is highly plausible that large distance behaviour of the total diagram is cut – off at large distances $|x^{(i)} - x^{(j)}|$ by the Wilson loops; by dimensional arguments it reduces to saying that the divergent integral $\int(x^{\frac{n}{2}-3})$, ($n \geq 6$) should be replaced by $\mu^{3-n/2}$, with $\mu \approx \sqrt{\sigma_s}$.

Correspondingly Eq.(54) is replaced in the framework of the NP background perturbation theory by

$$\tilde{J}^{(n)} \sim g^n V_3 T^{n/2} (\sqrt{\sigma_s})^{(3-n/2)} \quad (56)$$

Now at high $T, T \gg 2T_c$, as we discussed in chapter 4, the dimensional reduction mechanism is at work, which yields [7,19] $\sqrt{\sigma_s(T)} = cg^2(T)T$. Inserting this into (56) one gets

$$\tilde{J}^{(n)} \sim V_3 T^3 (cg^2(T))^3 \left(\frac{g}{c}\right)^{n/2} \quad (57)$$

Thus all diagrams with $n \geq 6$ are not IR diverging due to the Spatial string tension σ_s and their contribution to the free energy density can be written in the same form as in [26]

$$\frac{f(T)}{T} = \frac{\sum \tilde{J}^{(n)}}{V_3} = (g^2 T)^3 f_G + \dots \quad (58)$$

where $f_G = \sum_{n \geq 6} a_n \left(\frac{g}{c}\right)^{\frac{n-6}{2}}$ is given by the sum of diagrams with $n \geq 6$, and can be easily computed since $g(T)$, Eq.(27) is small for large T . Note however, that $c = c(g)$ as discussed, below Eq.(26).

Since our conclusion about the absence of the IR catastrophe is crucial for the theory, we present now another argument on convergence of $\tilde{J}^{(n)}$. To this end let us consider behaviour of the diagram, Fig.5., when one of the vertices, say $i = 1$, is far from all others. The diagram is actually longdistance limit of $3g$ glueball Green's function, since 3 gluons are produced at the vertex 1 in the gauge-invariant state with total angular momentum zero (this vertex is given by $L_6(a^3)$ term in (4)).

It is clear, that at large $|x^{(1)} - X|$, where X stands for the assembly of all other vertices, the diagram behaves as

$$\tilde{J}^{(n)}(x^{(i)}, x^{(2)} \dots) \sim e^{-M_{3g}(0^+) |x^{(1)} - X|} \quad (59)$$

where $M_{3g}(0^+)$ is the mass of the 3-gluon glueball in 3d, one expects that $M_{3g}(0^+) > 1 GeV$.

Now $M_{3g}(0^+) \sim \sqrt{\sigma_s} const$, where $const$ is some number of order of one. Thus again we obtain a cut-off in the x -space, which makes all integrals convergent and the estimate (57) remains true.

We note that nonperturbative background has created a situation equivalent (in the sense of convergence) to the appearance of the magnetic mass,

$$M_{magn} \approx \sqrt{\sigma_s} \sim g^2 T \quad (60)$$

Note however, that M_{magn} appears in a gauge-invariant manner, and not in Π_{ij} – self energy part of gluon. Rather it appears as in the constituent gluon model; if one defines $M_{gg} = 2M_{magn}$ (cf. Eq.(50)), then $M_{3g} \approx 3M_{magn}$, and each gluon propagator contributes in x -space asymptotics $G(x, y) \sim e^{-M_{magn}|x-y|}$. This asymptotics reproduces immediately (59) and (56). Also the Polyakov line correlator (48) containing the exchange of two gluon lines, decreases at large distances with screening mass $\mu = M_{gg} \approx 2M_{magn}$.

7 Conclusions

We have presented the formalism of QCD at $T > 0$, which contains NP field in the background, in the form of gauge-invariant correlators and perturbative expansion in this background.

It is shown how the Feynman-Schwinger representation (FSR) which proved to be very useful for $T = 0$, is modified for $T > 0$. In particular, all gluon and quark propagators are written as a sum of path integrals with winding paths in the 4th direction.

Using this formalism the structure of vacuum at $T < T_c$ and $T > T_c$ as well as the nature of deconfinement transition is clarified and T_c is computed in good agreement with lattice data. Moreover the spacial area law is shown to follow naturally from the structure of the vacuum at $T \geq T_c$. Using FSR the Hamiltonian for screening states of mesons and glueballs is derived, and screening masses and wave functions for lowest meson and glueball states are numerically computed.

Finally, it is demonstrated how the Infrared Catastrophe is eliminated due to the NP background and estimates of higher order diagrams for the free energy are given. All this exemplifies NP background field theory and in particular perturbation theory as a powerful instrument both for $T < T_c$ and for $T > T_c$.

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Figure captions

Fig. 1. The intraction measure, $(\varepsilon - 3p)/T^4$ as a function of temperature, from ref. [7].

Fig. 2. The normalized pressure p/p_{SB} and energy density $\varepsilon/\varepsilon_{SB}$ calculated from (16), (18) (p.back., e.back) and lattice calculations (p.latt., e.latt) as functions of T/Λ_L , from ref. [15].

Fig. 3. The ρ - meson (solid line) and glueball (dashed line) wave. functions for lowest Matsubara frequencies $vs\ r/a(a = 0.23fm)$ for $T = 210MeV$ from ref. [21]. The lattice data are from ref [23] for the same T .

Fig. 4. The screening masses for mesons (solid line) and glueballs (dashed line) as functions of temperature from ref. [21]. The lattice data are from ref. [5] (squares) and ref. [6] (triangle).

Fig. 5. The typical IR divergent diagram of hot QCD in the order g^n .

Fig.1, Fig.2, Fig.3, Fig.4, Fig.5,

$x_1, x_2, x_3, x_4, x_{n-1}, x_n$